The H-O neoclassical model (part 1)

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H-O Model: PRODUCTION STRUCTURE

Introduction
Neo-classical economics
General structure of the neo-classical model
Production functions
Cost minimization
Impact of wage rate and rental rate
Constant returns to scale
Conclusions
Paul Samuelson (1915 - )

Chapter 4 reviews the production structure of the neo-classical model

Introduction

Objectives / key terms

Production functions
Cost minimization
Constant returns to scale

Isoquants
Factor intensity
Unit costs

International Trade & the World Economy; © Charles van Marrewijk

1. The world economy

Part I

Explanations for trade

Classical
2. Opportunity costs
3. Comparative advantage

Part II

Neo-classical
4. Production structure
5. Factor prices
6. Production volume
7. Factor abundance

Part III

New trade
9. Imperfect competition
10. Intra-industry trade

Part IV

New interactions
14. Geographical economics
15. Multinationals
16. New goods, growth, and development

Chapter 4 reviews the production structure of the neo-classical model

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## Introduction

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### PRODUCTION STRUCTURE

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<td>Neo-classical economics</td>
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### Neo-classical economics

International trade based on differences in endowments results in:

- **Heckscher-Ohlin theorem (H-O)**
  - A country will export the final good which makes relatively intensive use of the relatively abundant factor of production

- **Factor price equalization (FPE)**
  - Trade in goods (which equalizes final goods prices) leads to equalization of factor prices

- **Stolper-Samuelson theorem (St-Sa)**
  - An increase in the price of a final good increases the reward to the factor used intensively in the production of that good and reduces the reward to the other factor

- **Rybczynski theorem (Ryb)**
  - An increase in the quantity of a factor of production at constant final goods prices leads to an increase in the production of the good using that factor intensively and a decreased production of the other good
### PRODUCTION STRUCTURE

**Neo-classical economics**

**General structure of the neo-classical model**

- Production possibility frontiers (PPF) and specialization in trade
- Production functions
- Cost minimization
- Impact of wage rate and rental rate
- Constant returns to scale
- Conclusions

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**International trade based on differences in endowments**

- Perfect competition
- Identical technology in the two countries
- Constant returns to scale; CRS
- Factor mobility between sectors, but not between countries
- No transport costs, no trade barriers
- Identical homothetic tastes in the two countries
- No factor-intensity reversal
- But differences in (relative) factor endowments

**Simplifications:**
- 2x2x2 model
- 2 factors of production; labor and capital (L and K)
- 2 goods; Food and Manufactures (F and M)
- 2 countries; Austria and Bolivia (A and B) or Home and Foreign
Formal structure of the neo-classical model

1. \[ a_{LM} w + a_{KM} r = P_M \]  
   perfect competition

2. \[ a_{LF} w + a_{KF} r = P_F \]  
   (or zero profits condition)

3. \[ a_{KM} M + a_{KF} F = K \]  
   budget constraint

4. \[ a_{LM} M + a_{LF} F = L \]  
   (or full employment condition)

General structure of the neo-classical model

The capital-stock per worker varies significantly between countries

NBER data for 1990 in 1985 $ (\times 1000)$
Production functions

\[ M = K_m^{\alpha_m} L_m^{1-\alpha_m}, \quad F = K_f^{\alpha_f} L_f^{1-\alpha_f} ; \quad 0 < \alpha_m, \alpha_f < 1 \]

An isoquant is the set of all efficient input combinations to produce a given amount of output.

Constant returns to scale

Suppose 5 labor and 15 capital can produce 10 M.
This is the isoquant associated with point A.
Under constant returns to scale a proportional increase in inputs leads to a proportional increase in output.
Suppose we increase K and L by 40%.
K from 15 to 21 and L from 5 to 7.
Then output also increases by 40% from M = 10 to M = 14.
Thus, the isoquant at point B is M = 14.
Production functions

The substitution possibilities between capital and labour in the neoclassical model are important (see the isoquant = 1 table below).

Table 4.1 Substitution possibilities ($\alpha_m = 0.5$)

<table>
<thead>
<tr>
<th>$L_m$</th>
<th>$K_m$</th>
<th>Extra capital</th>
<th>$L_m$</th>
<th>$K_m$</th>
<th>Extra capital</th>
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<tr>
<td>1.0</td>
<td>1.000</td>
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<td>0.5</td>
<td>2.828</td>
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<tr>
<td>0.9</td>
<td>1.171</td>
<td>0.171</td>
<td>0.4</td>
<td>3.953</td>
<td>1.124</td>
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<tr>
<td>0.8</td>
<td>1.398</td>
<td>0.226</td>
<td>0.3</td>
<td>6.086</td>
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<tr>
<td>0.7</td>
<td>1.707</td>
<td>0.310</td>
<td>0.2</td>
<td>11.180</td>
<td>5.095</td>
</tr>
<tr>
<td>0.6</td>
<td>2.152</td>
<td>0.444</td>
<td>0.1</td>
<td>31.623</td>
<td>20.442</td>
</tr>
</tbody>
</table>

Capital intensity $\alpha_m$ influences substitution.

\[ \alpha_m = 0.4 \]

\[ \alpha_m = 0.6 \]
Two industries with different factor intensities

\[ \begin{align*}
K_1 &> K_0 \\
L_1 &< L_0 \\
M_1 &< F_1 \\
\frac{K}{L} &> \frac{M}{K} \\
\frac{K}{L} &< \frac{F}{L} \\
\text{Slope} &= \frac{-w}{r}
\end{align*} \]

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Cost minimization; 1

An entrepreneur who wants to maximize profits can solve this problem in two steps:

1. Minimize production costs for any given output level
2. Using the outcome of the first problem: determine the optimal production level which will maximize profits.

We now address the first problem. Say the entrepreneur wants to produce at least 1 unit of good M with production function

\[ M = K_M^\alpha L_M^{1-\alpha}; \quad 0 < \alpha < 1 \]

The entrepreneur pays the wage rate \( w \) for the labor she uses and the rental rate \( r \) for the capital she uses, such that the costs are:

\[ wL_M + rK_M \]

The entrepreneur cannot influence wage rate \( w \) or rental rate \( r \).

Cost minimization; 2

The production function determines the isoquant \( X = 1 \).

The wage-rental ratio \( w/r \) determines the slope of an isocost line (= all combinations of \( K_M \) and \( L_M \) with the same level of production costs).

The entrepreneur can produce 1 unit of good X at points A and B (same cost level).

The objective is, however, to minimize the cost level.
Cost minimization; 3

Production costs decrease as the isocost line is closer to the origin. The cost minimizing input combination is therefore at the tangency point C, with \(K_M^*\) capital and \(L_M^*\) labor.

With the production function for good M these are:

\[
K_M^* = \left( \frac{\alpha w}{1 - \alpha r} \right)^{1-\alpha}
\]

\[
L_M^* = \left( \frac{\alpha w}{1 - \alpha r} \right)^{-\alpha}
\]

Cost minimization

The parameters \(\alpha_m, \alpha_f\) play an important role. They indicate the capital-intensity of the production process:

\[
\frac{K_m}{L_m} = \frac{\alpha_m}{1 - \alpha_m} \frac{w}{r}, \quad \frac{K_f}{L_f} = \frac{\alpha_f}{1 - \alpha_f} \frac{w}{r}
\]

We assume that the production of manufactures is more capital-intensive than the production of food, that is: \(\alpha_m > \alpha_f\).

Also note that:

\[
\text{total costs} = rK_m + wL_m
\]

\[
\alpha_m = \frac{\text{cost of capital}}{\text{total cost}} = \frac{rK_m}{rK_m + wL_m}
\]

So alpha also represents the share of costs paid to capital.
Cost minimization; changing relative factor prices

Clearly, if the wage-rental ratio changes, the cost minimizing input combination changes.

If the wage rate decreases, the optimal input combination changes from C to D; you use less capital and more labor.

The optimal capital-labor ratio therefore depends on the wage-rental ratio. It is:

\[ k_M \equiv \frac{K^*_M}{L^*_M} = \frac{\alpha}{1 - \alpha} \frac{w}{r} \]

It also depends on the capital intensity parameter alpha.

Cost minimization: simulation in the Marrewijk

<table>
<thead>
<tr>
<th>Exogenous variables</th>
<th>Baseline</th>
<th>Simulation</th>
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</thead>
<tbody>
<tr>
<td>Isoquant M =</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Total cost C =</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Rental rate (r)</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Wage rate (w)</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Alpha manufactures (αm)</td>
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<td>0.5</td>
</tr>
<tr>
<td>Capital input (Km)</td>
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<td>1.0</td>
</tr>
<tr>
<td>Labour input (Lm)</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Endogenous variables</th>
<th>Baseline</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>K/L ratio</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Total Production</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Total Cost</td>
<td>2.0</td>
<td>2.0</td>
</tr>
</tbody>
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General equilibrium analysis

Production functions & inputs

Graphic presentation of the production function of production of good F, requiring inputs of labor (L_F) and capital (K_F):

Graph a:
Q_F = f(L_F, T_F); production function of F with a given stock of capital (K_F); Q'_F = f(L_F, T'_F); production function of F with a larger stock of capital (K_F > K_F).
PPF & factor endowment

Changes in the production possibility frontier (PPF) resulting from changes in factor endowment \((K' > K^0)\) & \((L' > L^0)\).

Factor endowment \(\rightarrow\) relative factor and output prices

Assumption: Home Country relatively abundant in K in comparison to Foreign \((L/K) < (L^*/K^*)\) \(\rightarrow\)

domestic wages are relatively lower than rental rate in comparison to Foreign \(\rightarrow\)relative prices:

\[
\frac{P_M}{P_F} < \frac{P_M}{P_F}^{TOT} < \frac{P_M^*}{P_F^*}
\]

Liberalization of trade:
Home country exports M (capital-intensive) and imports good F (labor-intensive) \(\rightarrow\) At home relative price of M raises while price of good F decreases
Equilibrium in autarky: capital abundant country

Equilibrium in autarky: capital abundant country;
M: capital intensive good

\[ Q_M = K_M \]
\[ Q_F = K_F \]

\[ \text{Slope: } -(P_M/P_F) \]

QQ: PPF
UA: indifference curve in autarky
PC/PF: relative prices in autarky.

General equilibrium: gains from trade

Slope: -(P_M/P_F)

Value of production in world prices = Value of consumption in world prices, i.e.
\[ P_M^T Q_M + P_F^T Q_F = P_M^T K_M + P_F^T K_F \]
**Gains from trade: two countries**

**Notation:**
- $P_M/P_F$: relative prices in autarky;
- $P^T_M/P^T_F$: relative prices in the world economy (terms of trade);
- $A$: consumption and production equilibrium in autarky
- $A_Q$: production equilibrium in an open economy;
- $A_K$: consumption equilibrium in an open economy
- $DA_QA_K$ trade triangle at home ($DA_Q$ export of good $M$, a $DA_K$ import of good $F$)
- *: for foreign prices

**Gains from trade: basic relationships**

\[
\frac{P_M}{P_F} < \left( \frac{P^T_M}{P^T_F} \right) < \frac{P^*_M}{P^*_F} \tag{necessary condition for possible int'l trade}
\]

but it must be that:

\[
P^T_M Q_M + P^T_F Q_F = P^T_M K_M + P^T_F K_F
\]

Value of production in world prices = Value of consumption \(\Rightarrow\)

\[
P^T_M (Q_M - K_M) = P^T_F (K_F - Q_F) \tag{trade balance equilibrium}
\]

\[
\left( \frac{P^T_M}{P^T_F} \right) (Q_M - K_M) = (K_F - Q_F) \tag{trade triangles}
\]
Conclusions

Neo-classical trade model

- 2×2×2 structure (countries, goods, factors)
- perfect competition, constant returns to scale
- 4 main results (FPE, St-Sa, Ryb, HOS)
- different production factor intensities for goods
- different (relative) factor endowments for countries

Empirical simulations to Ch V. Marrevijk textbook are available at:
http://global.oup.com/uk/orc/busecon/economics/vanmarrewijk2e